Roundabout Trajectories and Intersections

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1 Key result

For a roundabout with four equally spaced exits, and four cars entering at different times and each passing three-quarters of the way round the roundabout, a formula giving the radial position r_p and angular position θ_p of car p at time t is

$$\begin{aligned} r_p &= (1.7-0.9\rho)-(19-21\rho)\left(\theta_p - p\frac{\pi}{2}\right)\exp\left(-\frac{5}{2}\left(\theta_p-\frac{\pi}{2}\right)^{2/5}\right), \\ \theta_p &= \omega(t-\tau_p)+p\pi/2, \end{aligned}$$

where

$$\frac{\theta_{ent}}{\omega}+\tau_p \leq t \leq \frac{\theta_{ex}}{\omega}+\tau_p.$$

Here, τ_p is the time that car p enters the roundabout. The parameter ρ relates to the size of the roundabout, $\rho = R_2/R_1$ where R_1 and R_2 are the outer and inner radii of the roundabout respectively. The speed v of the car is related to ω, $\omega \approx 2v/(1+\rho)$.

Figure 1 shows a plot of the four trajectories for a roundabout with $\rho = 1/2$. Note that θ is measured clockwise, unlike conventional polar coordinates which measure θ anticlockwise.

Figure 2 shows a plot of the trajectories when the roundabout is 'unwrapped'. Here, along the horizontal axis is plotted θ_p and along the vertical axis is plotted r_p. Since $\theta = 0$ is the same as $\theta = 360^o$, trajectories leaving at the right hand side re-enter on the left. Arrows have been added to indicate direction of travel. Note that the formula does not include the turn off the roundabout, indicated by the exit arrow.

The background to the formula is given below: a number of approximations have been used. There is considerable flexibility as to how complicated the formula looks depending on how many of these approximations are included.

The trajectories represent 'ideal' paths that cars would take around a roundabout. However, there are many intersection points and cars can only

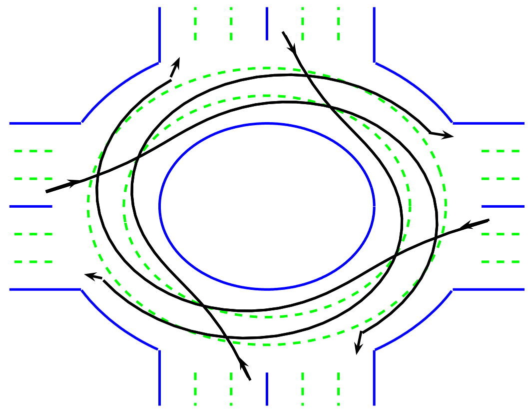

Figure 1: Trajectories on a roundabout with $\rho = 1/2$.

follow these paths if there is a precise relationship between their times of entry τ_p to the roundabout and the speed of cars on the roundabout, v. In order to avoid collisions, cars may have to follow an alternative trajectory. With the the cris-crossing of paths, good signalling and mirror technique are clearly important to optimize road safety.

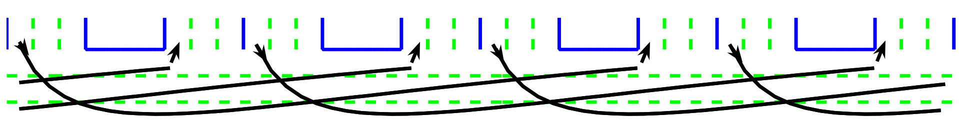

Figure 2: 'Unwrapped' trajectories for a roundabout with $\rho = 1/2$

2 Background

2.1 Single trajectory

Different functional forms were tested for a single trajectory with the aim of obtaining a balance between getting the right general features but with not too complex a formula. The best result was with a trajectory of the form

$$r = \alpha - \beta\theta \exp\left(-\gamma\theta^n\right),$$

where r measures the distance from the centre of the roundabout and θ measures the distance travelled around the roundabout, measured clockwise. The value of r has been scaled so that it is measured in units of the outer radius. This formula takes into account the movement of a car to the inner lane of the roundabout and then a spiralling out to reach the exit. The parameters α, β, γ and n are determined by choices of the position of entry θ_{ent}, the point of closest approach to the centre of the roundabout $(r, \theta) = (r_{min}, \theta_{min})$, and how rapidly the car spirals out. In terms of n

$$\begin{aligned} \gamma &= \frac{1}{n\theta_{min}^n}, \\ \beta &= \frac{1 - r_{min}}{\theta_{min} \exp\left(-\frac{1}{n}\right) - \theta_{ent} \exp\left(-\gamma\theta_{ent}^n\right)}. \\ \alpha &= r_{min} - \beta\theta_{min} \exp\left(-\frac{1}{n}\right). \end{aligned}$$

The choice for θ_{min} depends how rapidly a typical driver reaches the inner lane. Making the choice that $\theta_{min} = 1$ means that cars reach the inner lane by the time they have gone approximately 64^o around the roundabout. Sensible choices for the θ_{ent} and r_{min} and depend on how many lanes the roundabout has and the size of entry and exit roads. If we consider a three lane roundabout with three lanes entering and leaving, then reasonable choices would be

$$\begin{aligned} \theta_{ent} &= \frac{1}{6}(1 - \rho) \\ r_{min} &= \frac{1}{6}(1 + 5\rho). \end{aligned}$$

Here,

$$\rho = \frac{R_2}{R_1},$$

where R_1 and R_2 are the outer and inner radii of the roundabout respectively. Then we have,

$$\begin{aligned}
\gamma &= \frac{1}{n}, \\
\beta &= \frac{5(1-\rho)}{6\exp\left(-\frac{1}{n}\right) - (1-\rho)\exp\left(-\frac{1}{n}\left(\frac{1-\rho}{6}\right)^{n}\right)}, \\
\alpha &= \frac{1+5\rho}{6} + \beta\exp\left(-\frac{1}{n}\right).
\end{aligned}$$

This just leaves n. This is the parameter that is the predominant factor in determining how rapidly the cars spiral out. Expressions for n can be derived by, for example, fixing when the cars reach the centre of the middle lane or fixing the exit angle θ_{ex}. The resulting formulae are very messy. However, for roundabouts of sensible sizes a reasonable choice is to make $n = 2/5$. With this choice, we have,

$$\begin{aligned}
\gamma &= \frac{5}{2}, \\
\beta &= \frac{5(1-\rho)}{6\exp\left(-\frac{5}{2}\right) - (1-\rho)\exp\left(-\frac{5}{2}\left(\frac{1-\rho}{6}\right)^{5/2}\right)}, \\
\alpha &= \frac{36 - (1+5\rho)(1-\rho)\exp\left(\frac{5}{2}\left(1-\left(\frac{1-\rho}{6}\right)^{5/2}\right)\right)}{6 - (1-\rho)\exp\left(\frac{5}{2}\left(1-\left(\frac{1-\rho}{6}\right)^{5/2}\right)\right)}
\end{aligned}$$

This still makes the formula for a single trajectory complicated. Typical roundabouts only have a fairly small range of sensible values for ρ. With this is mind, a linear fit can be done to the formulae for α and β for $0.5 < \rho < 0.7$. This gives

$$\alpha = 1.7 - 0.9\rho,$$

and

$$\beta = 19 - 21\rho,$$

resulting in,

$$r = (1.7 - 0.9\rho) - (19 - 21\rho)\theta\exp\left(-\frac{5}{2}\theta^{2/5}\right).$$

2.2 Multiple trajectories

The formula for a single trajectory was for a car entering at an angle close to 0^o. Now if we suppose that we have four cars, one entering at each of four equally spaced sliproads then car p has trajectory

$$r_p = \alpha - \beta(\theta_p - p\pi/2) \exp\left(-\gamma((\theta_p - p * pi/2)^n\right),$$

where $\theta_{ent} + p\pi/2 \leq \theta_p \leq \theta_{ex} + p\pi/2$.

2.3 Multiple trajectories including time

Supposing that the cars have constant angular velocity ω, then θ_i can be related directly to the time. If all cars enter the roundabout at the same time, then,

$$\begin{aligned} r_p &= \alpha - \beta(\theta_p - p\pi/2) \exp\left(-\gamma((\theta_p - p\pi/2)^n\right), \\ \theta_p &= \omega t + p\pi/2, \end{aligned}$$

where t is time.

If cars enter at different times, the entry time for car p being given by τ_p then

$$\begin{aligned} r_p &= \alpha - \beta(\theta_p - p\pi/2) \exp\left(-\gamma((\theta_p - p\pi/2)^n\right), \\ \theta_p &= \omega(t - \tau_p) + p\pi/2. \end{aligned}$$